PERSPECTIVES on PROCEDURAL FLUENCY- TEACHING ALGORITHMS

Position 1: Teaching traditional algorithms or procedures should be a priority.	
What are some facts that support this position?	**What are some facts that counter this position?**
Easier for students to understand. Some topics, such as multiplying two negative numbers or dividing by a fraction are difficult to understand. The rules of the product of two negative numbers is a positive, and when dividing by a fraction, invert and multiply, are more easily remembered. (Skemp, 2006).	In the case of multiplying by a fraction, research suggests when students are encouraged to find new strategies for solving fraction division word problems, they do not choose the invert and multiply method. By not addressing the question before guiding students to a standard approach, students' number sense of what is reasonable may be undermined (Newton & Sands, 2012). The procedure cannot necessarily be applied to other contexts (Skemp, 2006). Algorithms may harm children's development of number sense by "unteaching" place value. (Kamii & Dominick, 1997).
Rewards are more immediate and more apparent. Students feel successful and confident (Skemp, 2006).	Real confidence and satisfaction come in building student understanding (Skemp, 2006).
You can get the correct answer more quickly. Even mathematicians use it (Skemp, 2006). Expedience is more important. If you take too long getting kids up to grade level, you risk losing them altogether (Bean, 2011)	This works only in familiar contexts, when students get to more difficult problems, the algorithm does not always work (Markovits & Sowder, 1994). The standard algorithm may not be the best method (Carroll & Porter, 1997).
Children who do not master the standard algorithms begin to have problems as early as Algebra I (Budd, Carson, Garelick, Klein, Milgram, Raimi, Schwartz, Stotsky, Williams, & Wilson, 2005).	The lack of understanding is behind the problems students experience in Algebra I. Algorithms force children to give up their own thinking and remember the next steps (Kamii & Dominick, 1997).
Students learn in a variety of ways. Teaching procedures is more efficient and discovery does not ensure proficiency. Successful programs use discovery for only a few topics, not all (Budd et al., 2005).	Student's natural tendencies do not always fit the standard algorithm. Students are encouraged to work together and discuss their solutions (Carroll & Porter, 1997). When students are allowed to struggle, they develop conceptual understanding that can be used connected to other problems (Hiebert & Grouws, 2007).
The starting point for the development of children's creativity and skills should be established concepts and algorithms (Budd et al., 2005). Students will only remember what they have extensively practiced.	Memorizing procedures does not help you understand advanced mathematics. This is a limited, disconnect way to teach mathematics.

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Large scale data from California and foreign countries show that children with learning disabilities do much better in more structured learning environments (Budd et al., 2005).	Students with disabilities can understand and incorporate more strategies for problem solving when cognitive strategy instruction is used (Krawec, Huang, Montague, Kressler & de Alba, 2013).
Applications are important and story problems make good motivators, but understanding should come from building the math for universal application (Budd et al., 2005).	Context often builds meaning and constrains errors in the same way that the use of manipulatives does. From Kindergarten students can create and solve number stories that reveal much about their understanding of mathematical situations (Carroll & Porter, 1997).

Position 2: Teaching concepts and allowing alternative algorithms should be a priority.	
What are some facts that support this position?	**What are some facts that counter this position?**
Students will be more adaptable to new tasks (Skemp, 2006).	Is too difficult for students to understand (Skemp, 2006). Sometimes the algorithm is more efficient and less complicated (Newton & Sands, 2012).
Procedures easier to remember when the conceptual basis is understood. There's higher retention with strategies that make sense (Skemp, 2006; Mark Markovtis & Sowder, 1994).	Takes too long to achieve. It does not matter to the majority of tests (Skemp, 2006).
More motivation for students when they understand the concept thus making the teacher's job easier (Skemp, 2006).	Students are more motivated and confident when they know the procedures, making the teacher's job easier (Ocken, 2001).
Conceptual understanding assists students in becoming agents of their own growth, actively seeking out new material and exploring new areas (Skemp, 2006).	Teachers need a deep understanding of mathematics concepts to teach mathematical concepts to students. (Ma, 1999)
Students will be able to make more connections that exist within the mathematics (Ma, 1999).	If a skill is needed in another subject before it can be understood, it is useless to force it (Skemp, 2006).
VADOE SOL and Common Core State Standards requires require students to justify and reason, problem solve, communicate, make connections and use mathematical representations.	
Conceptual understanding allows multiple entry points into a problem (Skemp, 2006). Students have more tools available and different kinds of problems are better solved by different strategies. (Carroll & Porter, 1997).	No awareness of the relationship between successive stages in understanding a concept as it becomes more sophisticated. Learner is dependent on outside guidance for learning each new way to get there (Skemp, 2006).

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Developing conceptual understanding results in good achievement, good retention, and a reduction in the amount of time children need to master computational skills (Carroll & Porter, 1997).	Procedures without connections take time to master (Ocken, 2001).
Students invent their own algorithms, gaining new understanding of operations and problem solving that enhances number sense and accuracy (Carroll & Porter, 1997).	Takes too much time, may not be accurate (Budd et al., 2005).
Children begin school capable of solving number problems using objects to model situations long before they have memorized facts or learned to use written symbols (Carroll & Porter, 1997).	Standard algorithms are more than just ways to get the answer, they have theoretical as well as practical significance (Ocken, 2001).
Algorithms may harm children's development of number sense by "unteaching" place value. Algorithms force children to give up their own thinking and remember the next steps (Kamii & Dominick, 1997).	Children need assistance to ensure they are using the right algorithm (Budd et al., 2005).

Summary and Recommendation:

The great debate known as the "math wars" has been fought for the past 25 years in schools, universities, homes and legislatures (Crary & Wilson, 2013; Bean 2011; Budd, Carson, Garelick, Klein, Milgram, Raimi, Schwartz, Stotshy, Williams, & Wilson, 2005). Teachers, mathematics professors, principals, parents, students, and policy makers continue to argue the best method for teaching mathematics to our children. Will learning that emphasizes mastering efficient mathematical algorithms, or learning that promotes sense making through conceptual understanding better prepare students for advanced mathematics?

Those who advocate teaching children mathematics procedurally stress the "fluency and accuracy that is needed for formal mathematical competency" (Ocken, 2001, p. 5). Proponents of teaching procedures using developed algorithms highlight the importance of students developing fundamental skill sets through practice. These educators believe that students are not merely participating in rote memorization without understanding (Winkler, 2002). Standard algorithms are both *reliable* and *efficient*

(Ocken, 2002, p. 5), better preparing students for success in advanced mathematics courses, where they will be required to quickly perform numerous calculations without thought (Ocken, 2002; Winkler, 2001).

When procedural instruction is provided, teaching is fast paced and teacher-directed. Familiar problems are more frequently used, producing error-free practice. No one can argue that students in calculus or physics courses need to perform complicated computations efficiently. Certainly, all people benefit from knowing multiplication facts, being able to calculate a tip at a restaurant or estimate purchases at the grocery store.

Advocates of teaching for conceptual understanding argue that it is more important for students to understand the concepts and processes surrounding the mathematics. Conceptual instruction allows students to develop understanding of the concepts, make connections, and struggle with important mathematical ideas in an intentional and conscious way (Hiebert & Grouws, 2007). Students are able to develop understanding by accessing prior knowledge, and add to their knowledge package. Research substantiates the value of teaching for understanding.

Studies by Markowits and Sowder (1994) found seventh grade students who were taught conceptually made connections to their prior knowledge, developed stronger rational number sense, and retained the concepts learned. Yang, Hsu, and Huang (2004) found similar results with sixth graders who were taught number sense when given unfamiliar problems to solve. These students were able to answer more questions correctly and had higher retention than the control group. In another study comparing United States and Chinese students, Liping Ma (1999) found the Chinese students, who were taught arithmetic conceptually, had a much stronger number sense and were more prepared for higher level mathematics than students from the United States, who were taught more procedurally.

Delaying formal instruction of algorithms is advantageous to student development of number sense and problem solving skills (Carroll, 2000). Kamii and Dominick (1997) found that forcing children

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to learn algorithms was actually harmful to developing student numerical thinking. Algorithms can "unteach" place value and "force students to give up their thinking" (Kamii and Dominick, 1997, p. 58). When teachers allow students to *invent* their own procedures that make sense of the situation, (Carroll, 2000, p. 110) mathematical reasoning is developed. Teachers should have a strong enough knowledge package to facilitate and ensure students are developing accurate procedures.

On one hand, it is vital for students to leave middle school with strong rational number sense. If they are only taught procedurally, this will be difficult to develop. On the other hand, as they leave middle school students must have a strong foundation that includes a mathematics toolkit ready to take on algebraic thinking that includes being able to perform numerical calculations effortlessly.

A hybrid model is truly the best approach. According to Miller, Stringfellow, Kaffar, Ferreira, and Manci (2011), conceptual learning combined with procedural are both required. Conceptual knowledge, strong number sense, and deep understanding allow students to generalize and apply their mathematics to unfamiliar problems. Procedural knowledge allows students to use steps of an algorithm in order to be efficient in producing an accurate result. Another type of knowledge, declarative knowledge, provides students with the ability to know mathematics facts immediately without having to think about a concept or procedure. Having fluency in all three of these knowledge areas creates strong mathematicians and increases the likelihood of student success in mathematics (Miller et al., 2011).

Teachers, administrators, parents and legislators should examine the recommendations of the organizations influencing mathematics instruction when deciding the best method for engaging, motivating, and preparing students mathematically. The National Council of Teachers of Mathematics (NCTM) advocates students learning algorithms along with the conceptual understanding, but "the teaching and learning of mathematics involves far more than memorizing procedures and applying algorithms" (Lim & Dillon, 2012, p. 316). The new Common Core State Standards and the Virginia

Department of Education have both adopted process standards that require students to justify, reason, problem solve, communicate, make connections, and use mathematical representations when solving problems.

Considering both research and professional organizations, middle school mathematics should incorporate many types of knowledge. Students should be presented with unique and challenging problems in order to develop number sense and problem solving skills. Teaching procedures allows students to become efficient and accurate, but students must be led to understand why the algorithms work, whether through discovery learning or direct instruction. Strong declarative knowledge provides students with efficiency, number sense, and confidence by knowing number facts including multiplication tables, perfect squares, and common percentages. Gaining all three types of knowledge in the middle school years will prepare students for algebraic thinking and make them stronger mathematicians.

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